New Elementary Mathematics 2

This test covers material taught in New Elementary Mathematics 2
(http://www.singaporemath.com/)

Calculators should not be used unless indicated by a C in front of the problem.

1. Simplify and express your answer in positive index form. Assume all variables represent non-zero integers.

   (a) $\dfrac{(-7)^9}{3^0 \times 3^{-8} \times (-7)^{-3}}$ [1]

   (b) $\dfrac{(m^7 n)^{-2}}{m^{-4} n^3}$ [1]

   (c) $(2x)^a \left(x^{-a}\right)$ [1]

2. Express in standard form A x 10^n correct to 3 significant figures. [2]

   $$\frac{41.2 \times 10^{-3} \times 1.21 \times 10^{-1}}{33 \times 10^{-7} \times 101 \times 10^5}$$

3. Expand the following expressions.

   (a) $(2x + y)(3x - 5y)$ [1]

   (b) $\left(\dfrac{x}{y^3} + xy\right)^2$ [2]

4. Factorize the following expressions.

   (a) $28x^5 - 58x^4 - 30x^3$ [1]

   (b) $3xy + yz - 9x^2 + z^2$ [1]

   (c) $9a^2 - (b + c)^2$ [1]

5. Solve for x.

   (a) $6x^3 - x^2 = x$ [1]

   (b) $6(x - 1)^2 = 16 - 8x$ [1]

   (c) $\dfrac{3}{x} = \dfrac{2x - 7}{5}$ [2]

6. Simplify the following.

(a) $\frac{a}{2}+\frac{a}{3}-\frac{3a}{8}$ [1]

(b) $\frac{5c}{a-4b}-\frac{2d}{4b-a}$ [1]

7. A bottle $\frac{3}{4}$ filled with liquid weighs 3 kg. The liquid alone weighs $\frac{3}{4}$ kg more than the empty bottle. If the bottle is completely filled, how much will the contents weigh? [4]

8. Express y in terms of k and a.

$y+3=\frac{2y+k}{a}$ [2]

9. Solve for x.

(a) $\frac{3(x-1)}{2}+\frac{2x}{3}=0$ [1]

(b) $\frac{6}{2x-5}=\frac{4}{x-3}$ [1]

(c) $\frac{2}{3}x-\left(x+\frac{1}{4}\right)=\frac{1}{12}(x+4)$ [1]

10. John bought some CDs for \$63. If the cost per CD was reduced by \$1, he would get 1 more CD by paying \$1 more. Find the cost of each CD. [3]

11. Hooke's law for an elastic spring states that the distance a spring stretches is proportional to the force applied. If a force of 150 newtons stretches a certain spring 8 cm, how much will a force of 400 newtons stretch the spring? [4]

12. The distance from the planet Pluto to Earth is 4.58×10^9 kilometers. Radio waves move at the speed of light, 3.00×10^5 km/s. Determine how long it will take to transmit radio signals from Pluto to Earth, giving your answer in standard form to 2 decimal places,

(a) in seconds. [3]

(b) in hours. [2]

13. How many liters of 60% acid solution must be mixed with a 75% acid solution to get 20 liters of a 72% solution? [4]

14. Calculate the following mentally. [2]
$101 \times 99 + 1 - 99^2$

15. (–3, 5), (2, 3), (2, –1) are three of the vertices of a rectangle.
    (a) Find the coordinates of the fourth vertex. [1]
    (b) Write down the equations of the lines which form the sides of the rectangle. [2]
    (c) Calculate the numerical value of the area. [1]

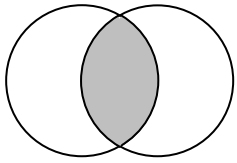
16. On a suitable grid, draw the graph of $y = x^2 - 1$. From the graph, estimate the values of x when y = 5, to one decimal place. [2]

17. Solve the following equations.
$2(x + 1) + (3y - 1) = 19$ [2]
$3(x + 2) - 2(y + 1) = 5$

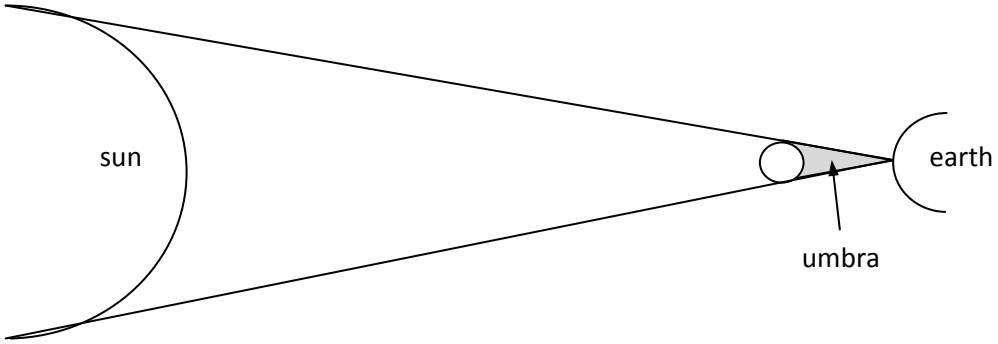
18. A boat takes 3 hours to go 48 km upstream. It can go 72 km downstream in the same time. Find the speed of the current and the speed of the boat in still water. [3]

19. Solve the inequality.
$1 - \frac{x}{3} \geq \frac{2}{3}\left(2x - \frac{3}{4}\right)$ [2]

20. Find the area of overlap of the two circles to 3 significant figures if radius of each circle is 1 cm. Take $\pi = 3.14$. [3]

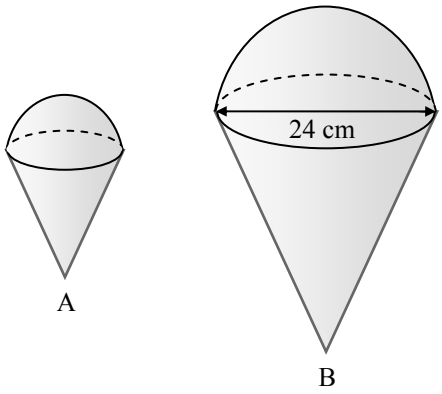


21. The sun has a diameter of about 1,380,000 km with a maximum distance from the earth's surface of about 151,000,000 km. The moon has a smaller diameter of about 3,450 km. For a solar eclipse to occur, the moon must pass between the earth and the sun. The moon must also be close enough to the earth for the moon's umbra (shadow) to reach the surface of the earth. Calculate the maximum distance that the moon can be from the earth and still have a total solar eclipse. [4]

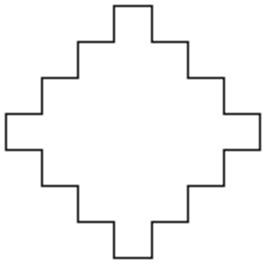


22. The figure below shows a pair of similar solids, each made up of a cone and a hemisphere. The volume of solid B is 6,032 cm^3. If the volume of solid B is 8 times the volume of solid A, find the surface area of solid A to 3 significant figures. [4]

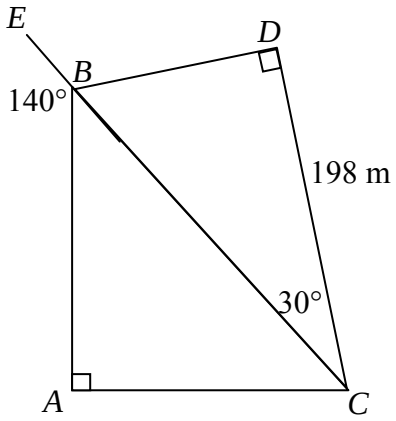
Take $\pi = 3.14$. $V_{cone} = \frac{1}{3}bh$, $V_{sphere} = \frac{4}{3}\pi r^3$, $S_{cone} = \pi rl$, $S_{sphere} = 4\pi r^2$.



23. All sides of the following figure have the same length. The perimeter in cm and the area in cm^2 are the same. What is the perimeter of the figure in cm? [3]



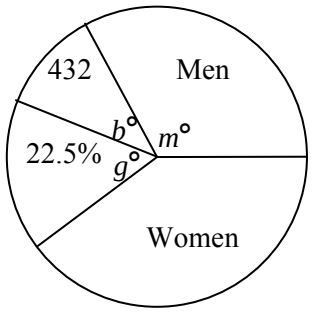
C 24. The following figure is a drawing of a piece of land. It is not drawn to scale. EBC is a straight line, $\angle EBA = 140^o$ and $\angle BCD = 30^o$. Find the length of the property edge AB to the nearest meter. [4]



C 25. Peter needs to know the height of a tree. From a given point on the ground he finds that the angle of elevation to the top is 50^o. He then moves back 28 m. From the second point, the angle of elevation to the top of the tree is 30^o. Find the height of the tree to the nearest cm. [3]

26. Find the coordinates of an image of (–1, 4) under
    (a) a translation which moves the point 3 units in the x direction and -2 units in the y direction, [2]
    (b) an anti-clockwise rotation of 90^o, [2]
    (c) and a reflection about the line $y = 3$. [2]

27. The pie chart (angles are not drawn to scale) represents the distribution of a population of 2,880 people in a small town. There are 432 boys, 22.5% of the population is girls, and there are twice as many women as men.



(a) Find b, g, and m. [2]

(b) Find the number of females in the town. [2]

28. The temperatures in degree Celsius (°C) each day over an eight-week period were as follows:

32, 28, 30, 32, 31, 29, 31, 27, 28, 30, 31, 31, 32, 31,
33, 30, 29, 27, 31, 31, 32, 30, 31, 31, 30, 32, 32, 32,
30, 29, 28, 28, 29, 27, 30, 32, 32, 31, 28, 32, 33, 32,
33, 30, 28, 27, 28, 29, 27, 29, 28, 30, 31, 32, 33, 32

(a) Construct a frequency chart. [1]

(b) Draw a histogram to represent the results. [1]

(c) What is the most common temperature? [1]

(d) Find the fraction of days in which the temperatures were 30° C or lower. [1]

29. A six sided die is thrown 24 times. The results are shown in the table below.

Number on die	1	2	3	4	5	6
Frequency	7	3	3	5	2	4

(a) What is the mode? [1]

(b) What is the median? [1]

(c) The die is thrown one more time. Find the number shown on the die if the mean of the total throws is 3.2. [2]

30. (a) Find a and b for the following identity.
$(4x + 1)(3x – 2) = ax^2 – bx – 2$ [2]

(b) Express x in terms of z if y = 3x + 6 and y = $\frac{z+5}{3}$. [2]

Answer Key

1. (a) $7^{12} \times 3^8$ (b) $\frac{1}{m^{10}n^5}$ (c) 2^a
2. 1.50×10^{-4}
3. (a) $6x^2 - 7xy - 5y^2$ (b) $\frac{x^2}{y^6} + \frac{2x^2}{y^2} + x^2y^2$
4. (a) $2x^3(7x + 3)(2x - 5)$ (b) $(y - 3x + z)(3x + z)$ (c) $(3a + b + c)(3a - b - c)$
5. (a) $0, \frac{1}{2}, -\frac{1}{3}$ (b) $-1, 1\frac{2}{3}$ (c) $5, -1\frac{1}{2}$
6. (a) $\frac{11a}{24}$ (b) $\frac{5c+2d}{a-4b}$
7. $2\frac{1}{2}$ kg
8. $y = \frac{k-3a}{a-2}$
9. (a) $\frac{9}{13}$ (b) 1 (c) $-1\frac{2}{5}$
10. \$9
11. $21\frac{1}{3}$ cm
12. (a) 1.53×10^4 s (b) 4.24 h
13. 4 liters
14. 199
15. (–3, –1) (b) $x = -3, x = 2, y = -1, y = 5$ (c)30
16. 2.5 and -2.5
17. $x = 3, y = 4$
18. Speed of current = 4 km/h Speed of boat = 20 km/h
19. $x \leq \frac{9}{10}$
20. 1.23 cm^2
21. 377,500 km
22. 415 cm^2
23. 31.36 cm
24. 175 m
25. 31.36 m
26. (a) (2, 2) (b) (-4, -1) (c) (-1, 2)
27. (a) $b = 54, g = 81, m = 75$ (b) 1848
28. (c) 32° C (b) $\frac{1}{2}$
29. (a) 1 (b) 3 (c) 4
30. (a) $a = 12, b = -5$ (b) $x = \frac{z-13}{9}$