

Week	Topic / Objectives	Strategies	Activities	Resources	Remarks / Update
Sec 4 Term 1 Week 4	<p>CHAPTER 13 TANGENTS, NORMALS AND RATES OF CHANGE</p> <p>13.1 Tangents And Normals</p> <p>A. Tangents</p> <p>B. Normals</p> <ul style="list-style-type: none"> • Find the equation of the tangent to a curve at a point • Define the normal to a curve at a point and find its equation 	<p>Recalling the concepts of gradients of parallel and perpendicular lines, and the method of finding the equation of a straight line</p> <p>Applying differentiation to find the gradient of the tangent to a curve at a point and hence finding its equation</p> <p>Illustrating the relationship between the tangent to a curve at a point and the normal to the curve at the same point, and hence finding its equation</p>		<p>p.303-308 Textbook</p> <p>I-Book Teacher's Guide</p>	
Sec 4 Term 1 Week 5	<p>13.2 Rates Of Change</p> <ul style="list-style-type: none"> • Understand that the derivative can be regarded as the rate of change • Solve problems involving rates of change <p>13.3 Connected Rates Of Change</p> <ul style="list-style-type: none"> • Apply differentiation to problems involving connected rates of change 	<p>Exploring the interpretation of the derivative of a function as the rate of change of a variable</p> <p>Illustrating some common real-life examples of rates of change</p> <p>Demonstrating the relationship between the rates of change of two related quantities with respect to time</p> <p>Applying differentiation (Chain Rule) to solve problems involving connected rates of change</p>	<p>p.308 Class Activity 1</p>	<p>p.308-312 Textbook</p> <p>p.312-317 Textbook</p> <p>I-Book Teacher's Guide</p>	

Chapter 13 Tangents, Normals And Rates Of Change

Suggested Approach

In Chapter 12, students have learnt that the derivative of a function $f(x)$ at $x = c$ is the gradient of the tangent to the curve at the point, where $x = c$, on the curve $y = f(x)$. We consolidate this idea here by finding the equations of the tangent and the normal to the curve at $x = c$. It would be useful if students can sketch the graph of the function and its tangent and/or its normal using a graphing software. This would allow them to visualise the behaviour of the function and the variation of the tangents at different points on the curve.

Students should recognise that the derivative of a function may be regarded as an instantaneous rate of change. When a quantity y is a function of time t , then the derivative $\frac{dy}{dt}$ is the rate of change of y with respect to t . The velocity of an object is a good example for illustrating this idea.

When two related quantities x and y are both functions of time t , we have the issue of related rates of change. To illustrate this concept, teachers may use a table of corresponding values of x , y , $\frac{dx}{dt}$ and $\frac{dy}{dt}$. Teachers may also use a spreadsheet program, which can facilitate the manipulation of values.

Section 13.1 Tangents and Normals

The geometric meaning of the derivative of a function $f(x)$ is the gradient of the tangent to the curve $y = f(x)$ at a point. By evaluating the value of the gradient, $f'(c)$, we can find the equation of the tangent at the point where $x = c$. When the gradient is undefined, the tangent is the vertical line $x = c$.

Since the normal to a curve at a point is perpendicular to the tangent to the curve at that point, the gradient of the normal at a point $(c, f(c))$ is given by $-\frac{1}{f'(c)}$, provided $f'(c) \neq 0$. Similarly, the equation of a normal can be found.

In solving such problems, students are encouraged to sketch the graphs of the curve and the tangent concerned.

Section 13.2 Rates of Change

Students should note that when y is a function of time t , t is the independent variable of the function.

The velocity of an object is the rate of change of displacement over time.

Section 13.3 Connected Rates of Change

When y is a function of x , that is, $y = f(x)$, the rates of change of x and y are related by the equation $\frac{dy}{dt} = \frac{d}{dx} [f(x)] \times \frac{dx}{dt}$. For connected rates of change problems, typical examples include cases such as water leaking from an inverted cone and the rate of change of the length of a man's shadow when the man is walking away from a street lamp.



TANGENTS, NORMALS AND RATES OF CHANGE

Class Activity 1

Objective: To understand that the derivative of a function can be interpreted as the rate of change of a variable.

A car accelerates from rest along a straight road. Its displacement y m from the starting point at time t seconds is given by $y = f(t) = \frac{3}{2}t^2$. Let us find the velocity of the car when $t = 5$.

1. Copy and complete the following table, where $\delta y = f(5 + \delta t) - f(5)$.

δt	1	0.5	0.2	0.1	0.001
$5 + \delta t$	6	5.5	5.2	5.1	5.001
$f(5 + \delta t)$	54	45.375	40.56	39.015	37.5150015
δy	16.5	7.875	3.06	1.515	0.0150015
$\frac{\delta y}{\delta t}$	16.5	15.75	15.3	15.15	15.0015

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2. In **Question 1**, what does $\frac{\delta y}{\delta t}$ represent?

$\frac{\delta y}{\delta t}$ represents the average velocity of the car during the period $t = 5$ to $t = 5 + \delta t$.

3. Suggest the value of $\lim_{\delta t \rightarrow 0} \frac{\delta y}{\delta t}$ in **Question 1**. What does it represent?

$\lim_{\delta t \rightarrow 0} \frac{\delta y}{\delta t} = 15$. It represents the instantaneous velocity of the car when $t = 5$.

4. Express $\lim_{\delta t \rightarrow 0} \frac{f(t + \delta t) - f(t)}{\delta t}$ in differentiation notation.

$\lim_{\delta t \rightarrow 0} \frac{f(t + \delta t) - f(t)}{\delta t} = \frac{dy}{dt}$

Level 3

9. A stone is thrown vertically upwards. Its height, h m, above the ground at time t seconds is given by

$$h = 20t - 5t^2,$$

where $0 \leq t \leq T$, and T is the time of flight of the stone.

- Find the value of T .
- Draw the graph of h against t .
- Find the interval of the time t for which the velocity is positive.

Solution

(a) $h = 20t - 5t^2$

When the stone reaches the ground, $h = 0$ and $t = T$.

$$\therefore 20T - 5T^2 = 0$$

$$5T(4 - T) = 0$$

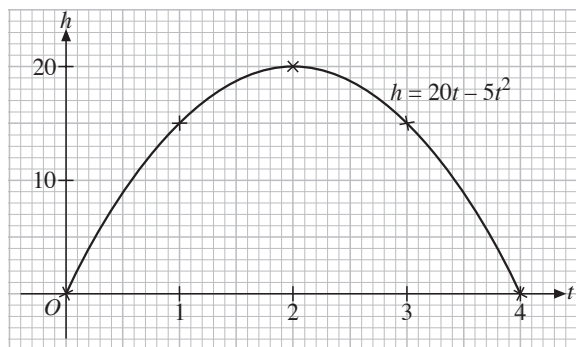
$$T = 4 \quad \text{or} \quad 0 \quad (\text{rejected})$$

$$\therefore T = 4$$

(b)

t	0	1	2	3	4
$h = 20t - 5t^2$	0	15	20	15	0

The diagram below shows the graph of h against t .



(c) Velocity, $v = \frac{dh}{dt}$
 $= (20 - 10t)$ m/s

For which $v > 0$,

$$20 - 10t > 0$$

$$10t < 20$$

$$t < 2$$

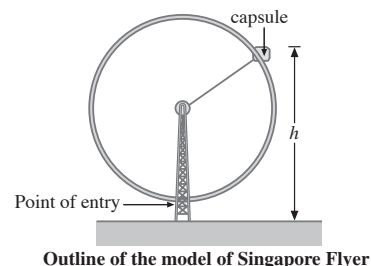
\therefore the required interval of t is $0 \leq t < 2$.

10. A tourist enters a capsule of the Singapore Flyer when the capsule is at the lowest point above the ground. The height, h m, of the capsule above the ground after t minutes is modelled by

$$h = 90 - 75 \cos \frac{2\pi t}{30}.$$

- (a) Find the time T minutes for one complete revolution of the capsule.

- Find the minimum height and the maximum height of the capsule above the ground.
- Draw the graph of h against t for $0 \leq t \leq T$.
- Find two possible values of t , where $0 \leq t \leq T$, when the capsule is 120 m above the ground.
- Find the rate of change of h when $t = 12.5$.

**Solution**

- (a) Time taken for one complete revolution, T
 $=$ period of the function

$$= \frac{2\pi}{\frac{2\pi}{30}}$$

$$= 30 \text{ minutes}$$

- (b) $h = 90 - 75 \cos \frac{2\pi t}{30}$
- Minimum height $= 90 - 75(1) = 15$ m when $\cos \frac{2\pi t}{30} = 1$
- Maximum height $= 90 - 75(-1) = 165$ m when $\cos \frac{2\pi t}{30} = -1$

(c) $h = 90 - 75 \cos \frac{2\pi t}{30}$

t	0	5	10	15	20	25	30
h	15	52.5	127.5	165	127.5	52.5	15

The diagram below shows the graph of h against t .

