

POLYNOMIALS AND PARTIAL FRACTIONS

3



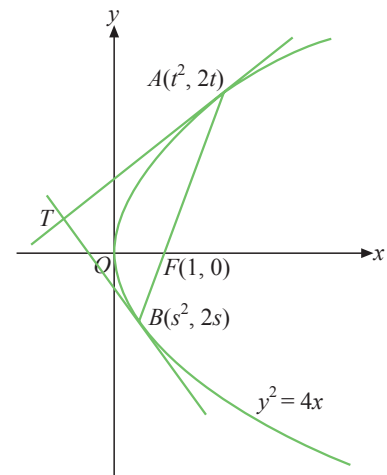
Level 1

- Expand and simplify the following.
 - $(3x^2 - 6x + 7) + (2x^2 + 5x - 3)$
 - $(x^3 - 4x + 8) - (9x^3 + 8x^2 - 7x + 1)$
 - $(2x^2 - x + 5)(x + 2)$
 - $(3x - 7)(2x^2 + 6x + 1)$
 - $(1 - 8x - x^2)(5 - 4x)$
 - $(2x^2 - x + 3)(x^2 + 4x - 7)$
 - $(3x^3 + 6x - 1)(x^2 - 5)$
 - $(3 - x + 2x^2)(5 - 4x - x^2)$
- Find the coefficient of
 - x in $(3x^2 - 4x + 7)(2x - 5)$,
 - t^2 in $(t^2 + 4t - 6)(3t^2 - 2t + 1)$,
 - y^3 in $(5 - 2y - 4y^3)(1 - y^2 - 3y^3)$,
 - z^4 in $(3z - 6 - 5z^3)(7z + 1 + 9z^3)$,
 - x^2 in $(2x^2 + 3x - 1)^2$,
 - t in $(3t - 4)^3$.
- Find the quotient and remainder in each of the following divisions.
 - $3x^3 - 2x^2 + 2x - 1$ divided by $x + 1$
 - $2x^3 - 3x^2 + x + 5$ divided by $x - 2$
 - $2y^3 + 3y^2 - 17y - 6$ divided by $2y - 3$
 - $27y^3 + 64$ divided by $3y + 4$
 - $5x^3 - 3x + 4x^2 + 8$ divided by $x^2 + x + 2$
 - $2x^4 - 9x^3 + 4x - 20$ divided by $x^2 - 3$
 - $1 + 5x - 2x^2 + 8x^4$ divided by $x^2 + 1 - 3x$
 - $13t - 3 + 14t^3 - 25t^2 + 10t^4$ divided by $-3 + 4t + 2t^2$
- Find the remainder in each of the following divisions.

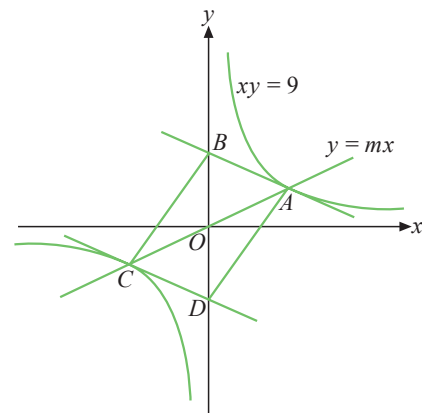
(a) $2x^3 - 3x^2 + 4x - 5$ divided by $x + 1$	(b) $x^3 + 4x^2 + 7x + 6$ divided by $x - 2$
(c) $3x^3 - 8x - 7$ divided by $x + 3$	(d) $4x^3 + 9x^2 - 3x + 1$ divided by $2x - 1$
(e) $9x^3 + 6x^2 - 8x$ divided by $3x + 2$	(f) $5x^4 - 10x^3 - 7x + 4$ divided by $5x + 3$


Level 3

21. (a) If $P(x_0, y_0)$ is a point on the curve $y^2 = 4x$, show that the gradient of the tangent to the curve at P is equal to $\frac{2}{y_0}$.
- (b) (i) Show that $A(t^2, 2t)$ is a point on the curve $y^2 = 4x$, where t is a real number.
(ii) If $t \neq 0$, find the equation of the tangent to the curve at A .
- (c) $F(1, 0)$ is a fixed point. $A(t^2, 2t)$ and $B(s^2, 2s)$ are two points on the curve $y^2 = 4x$ such that A, F and B are on the same straight line (refer to the diagram).
(i) Show that $st = -1$.
(ii) If TA and TB are the tangents to the curve at A and B respectively, show that $\angle ATB = 90^\circ$.

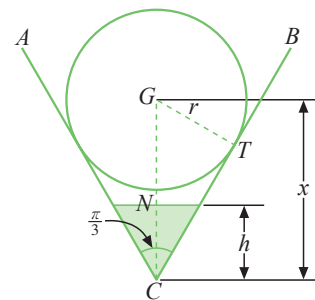


22. In the diagram, A and C are the points of intersection of the curve $xy = 9$ and the line $y = mx$, where $m > 0$. The tangents to the curve at A and C meet the y -axis at B and D respectively.
- (a) Express the coordinates of A and C in terms of m .
(b) Express the coordinates of B and D in terms of m .
(c) Show that $ABCD$ is a parallelogram.
(d) If the rate of change of m with respect to time t is 4 units/s, find the rate of change of the area of the parallelogram $ABCD$ with respect to time.



23. The equation of a curve is $y = \sqrt{x^2 - 5x + 4}$.
- (a) Find the range of values of x for which the curve is undefined.
(b) Find the points at which the curve cuts the x -axis and the y -axis.
(c) Find the equation of the normal to the curve at the point where $x = 5$.

24. The diagram shows the vertical cross-section ABC of a vessel in the shape of a right circular cone, where $\angle ACB = \frac{\pi}{3}$ rad., and a piece of spherical ice with centre G in it. The piece of ice melts at a constant rate of 2π cm³/min and it is always spherical in shape. When the radius of the ice is r cm, its centre G is x cm above C , the depth of water in the vessel is h cm and the volume of water is V cm³.
- (a) When $h = 4$, find the rate of increase of the water level.
(b) When $r = 5$, find the rate of change of the distance CG .



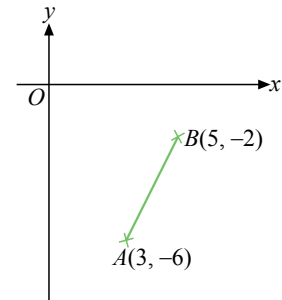
PAPER 2

Time allowed: $2\frac{1}{2}$ hours

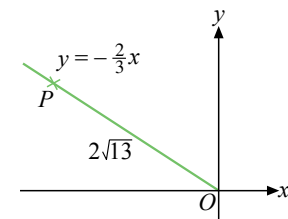
Answer all the questions.

1. If $(p - 4\sqrt{7})(2 + q\sqrt{7}) = 146 - 23\sqrt{7}$, where p and q are integers, find the values of p and q .

2. Two points $A(3, -6)$ and $B(5, -2)$ are given. Find
 (a) the equation of the perpendicular bisector of AB ,
 (b) the equation of the circle which passes through A and B , and has its centre on the y -axis.



3. In the diagram, P is a point in the second quadrant. It is on the line $y = -\frac{2}{3}x$ and is at a distance $2\sqrt{13}$ units from the origin.



- (a) Find the coordinates of P .
 (b) Find the equation of the circle which passes through P and has its centre at $G(-3, 5)$.
 (c) Is OP a tangent to the circle in (b)? Explain briefly.

4. $L_1 : 2x - 5y + 10 = 0$ and $L_2 : 4x + ky - 8 = 0$, where k is a constant, are two parallel lines.

- (a) Find the value of $\begin{vmatrix} 2 & -5 \\ 4 & k \end{vmatrix}$.
 (b) Find the value of k .
 (c) Find the area of the triangle bounded by L_1 and the two axes.

5. Let $\mathbf{A} = \begin{pmatrix} 3 & 6 \\ 2 & 5 \end{pmatrix}$.

- (a) Find $(\mathbf{A}^{-1})^2$.
 (b) Is $(\mathbf{A}^{-1})^2 = (\mathbf{A}^2)^{-1}$? Explain briefly.
 (c) Find the matrix \mathbf{B} if $\mathbf{A}^2 \mathbf{B} = \begin{pmatrix} 1 & 0 \\ -4 & 7 \end{pmatrix}$.

6. In the diagram, chords AC and BD intersect at E , M is the midpoint of BC , F is a point on AC such that $MF \parallel BD$. $AE = 6$ cm, $BE = 9$ cm and $DE = 12$ cm. Find the lengths of MF and CF .

