25. The numerator of a fraction is 5 less than its denominator. The sum of the fraction and its reciprocal is $2\frac{25}{104}$.

(a) If the denominator of the fraction is x, write the sum of the fraction and its reciprocal in terms of x.

(b) Form an equation in terms of x and show that it can be reduced to $x^2 - 5x - 104 = 0$. Hence, solve the equation to find the value(s) of x.

Solution:

(a) The fraction $= \frac{x-5}{x}$

Its reciprocal $= \frac{x}{x-5}$

Sum of the fraction and its reciprocal $= \frac{x-5}{x} + \frac{x}{x-5}$

$$= \frac{(x-5)^2 + x^2}{x(x-5)}$$

$$= \frac{2x^2 - 10x + 25}{x(x-5)}$$

(b) $\frac{2x^2 - 10x + 25}{x(x-5)} = 2\frac{25}{104}$

$\frac{2x^2 - 10x + 25}{x(x-5)} = \frac{233}{104}$

$104(2x^2 - 10x + 25) = 233(x^2 - 5x)$

$233x^2 - 208x^2 - 1165x + 1040x - 2600 = 0$

$25x^2 - 125x - 2600 = 0$

$x^2 - 5x - 104 = 0$

$(x+8)(x-13) = 0$

$x = -8$ (rejected) or $x = 13$

$\therefore x = 13$

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