

(g) (i) Let $S = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

$$\text{Then } SA = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \\ = \begin{pmatrix} 5a_1 & 5b_1 & 5c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

$$\therefore S = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

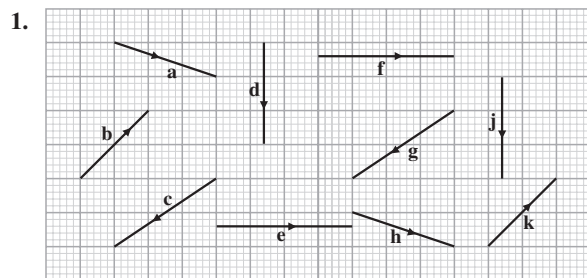
(ii) Let $S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$.

$$\text{Then } AS = \begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \\ = \begin{pmatrix} a_1 - 2c_1 & b_1 & c_1 \\ a_2 - 2c_2 & b_2 & c_2 \\ a_3 - 2c_3 & b_3 & c_3 \end{pmatrix}$$

$$\therefore S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}$$

Chapter 4 Vectors In Two Dimensions

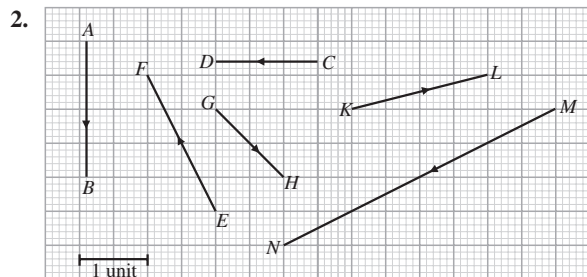
Basic Practice



The diagram above shows 10 vectors. Identify and write down the vectors that are equal.

Solution

$$\begin{aligned} a &= h \\ b &= k \\ c &= g \\ d &= j \\ e &= f \end{aligned}$$



Refer to the diagram above. Find the magnitude and direction of each of the following vectors.

- (a) \overrightarrow{AB} (b) \overrightarrow{CD}
 (c) \overrightarrow{EF} (d) \overrightarrow{GH}
 (e) \overrightarrow{KL} (f) \overrightarrow{MN}

Solution

(a) $|\overrightarrow{AB}| = 2$ units

\overrightarrow{AB} is along the negative direction of the y-axis.

(b) $|\overrightarrow{CD}| = 1\frac{1}{2}$ units

\overrightarrow{CD} is along the negative direction of the x-axis.

(c) $|\overrightarrow{EF}| = \sqrt{1^2 + 2^2}$

$$= \sqrt{5}$$

$$= 2.24 \text{ units (correct to 3 sig. fig.)}$$

The direction of \overrightarrow{EF} is upwards along a line with gradient -2 .

(d) $|\overrightarrow{GH}| = \sqrt{1^2 + 1^2}$

$$= \sqrt{2}$$

$$= 1.41 \text{ units (correct to 3 sig. fig.)}$$

The direction of \overrightarrow{GH} is downwards along a line with gradient -1 .

(e) $|\overrightarrow{KL}| = \sqrt{2^2 + \left(\frac{1}{2}\right)^2}$

$$= \sqrt{\frac{17}{4}}$$

$$= 2.06 \text{ units (correct to 3 sig. fig.)}$$

The direction of \overrightarrow{KL} is upwards along a line with gradient $\frac{1}{4}$.

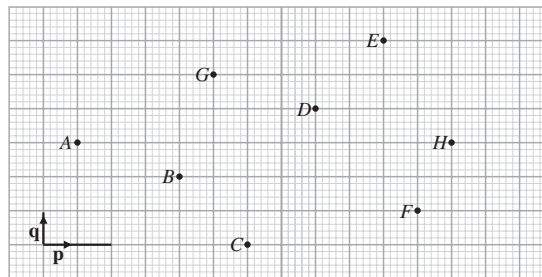
(f) $|\overrightarrow{MN}| = \sqrt{2^2 + 4^2}$

$$= \sqrt{20}$$

$$= 4.47 \text{ units (correct to 3 sig. fig.)}$$

The direction of \overrightarrow{MN} is downwards along a line with gradient $\frac{1}{2}$.

3. The points A to H, and the vectors \mathbf{p} and \mathbf{q} are shown on the diagram below.



Express each of the following vectors in terms of \mathbf{p} and/or \mathbf{q} .

- (a) \overrightarrow{AH} (b) \overrightarrow{AG}
 (c) \overrightarrow{DE} (d) \overrightarrow{GD}
 (e) \overrightarrow{EH} (f) \overrightarrow{FD}
 (g) \overrightarrow{CA} (h) \overrightarrow{DB}

Solution

(a) $\overrightarrow{AH} = 5\frac{1}{2}\mathbf{p}$

(b) $\overrightarrow{AG} = 2\mathbf{p} + 2\mathbf{q}$

Challenging Practice

23. (a) The cost price of a car is \$ x . A customer may pay \$ y in cash for the car. In this case, the profit made by the car dealer will be $13\frac{1}{3}\%$ of the mean of the cost price and the cash price.
- Show that $x : y = 7 : 8$.
 - Hence, find the corresponding profit as a percentage of the cost price.
- (b) The car may also be bought by paying a downpayment of \$ p and monthly instalments of \$ q over 60 months. Suppose that the cash price of the car is \$60 000 and the flat rate of interest is 4% p.a.
- Show that an equation in p and q is $p + 50q = 60\ 000$.
 - The total amount paid for downpayment and monthly instalments after 1 year is \$22 000. Show that another equation in p and q is $p + 12q = 22\ 000$.
 - Solve the simultaneous equations in (i) and (ii).
 - Hence, find the total amount payable if the car is bought by hire-purchase.

Solution

- (a) (i) Profit made = $\$(y - x)$
Mean of cost price and cash price = $\$\left(\frac{x + y}{2}\right)$
 $\therefore y - x = \frac{13\frac{1}{3}}{100}\left(\frac{x + y}{2}\right)$
 $y - x = \frac{40}{3}\left(\frac{x + y}{200}\right)$
 $200(y - x) = \frac{40(x + y)}{3}$
 $15y - 15x = x + y$
 $14y = 16x$
 $\frac{x}{y} = \frac{14}{16}$
 $= \frac{7}{8}$
 $\therefore x : y = 7 : 8$
- (ii) Profit as a percentage of the cost price
 $= \frac{1}{7} \times 100\%$
 $= 14\frac{2}{7}\%$ or 14.3% (correct to 3 sig. fig.)
- (b) (i) Total payment by hire-purchase = $\$(p + 60q)$
Interest = $\$(p + 60q - 60\ 000)$
By the formula,
interest = $\$\left[(60\ 000 - p) \times \frac{4}{100} \times \frac{60}{12}\right]$
 $= \$\left[\frac{1}{5}(60\ 000 - p)\right]$
 $p + 60q - 60\ 000 = \frac{1}{5}(60\ 000 - p)$
 $5p + 300q - 300\ 000 = 60\ 000 - p$
 $6p + 300q = 360\ 000$
 $p + 50q = 60\ 000$ ----- (1)
- (ii) Monthly instalments paid after one year = $12 \times \$q$
 $= \$12q$
 $p + 12q = 22\ 000$ ----- (2)

- (iii) (1) - (2): $38q = 38\ 000$
 $q = 1000$
Substituting $q = 1000$ into (2),
 $p + 12 \times 1000 = 22\ 000$
 $p = 10\ 000$
 $\therefore p = 10\ 000$ and $q = 1000$
- (iv) Total amount payable for hire-purchase
 $= \$(10\ 000 + 60 \times 1000)$
 $= \$70\ 000$

24. Mr Wee repaid a loan of \$10 000 at 10% p.a. compounded annually in 3 annual instalments of \$ w , \$2 w and \$3 w respectively.
- Express in terms of w , the balance of Mr Wee's loan just after his
 - first repayment,
 - second repayment.
 - (i) Form an equation in w and solve it. Give your answer correct to 2 decimal places.
(ii) Hence, express the total interest paid as a percentage of the original loan.
 - Mr Wee repaid his loan with progressively higher annual repayments. Give a possible reason for his action.

Solution

- (a) (i) Balance of loan after first repayment
 $= \$\left[10\ 000\left(1 + \frac{10}{100}\right) - w\right]$
 $= \$(11\ 000 - w)$
- (ii) Balance of loan after second repayment
 $= \$\left[(11\ 000 - w)\left(1 + \frac{10}{100}\right) - 2w\right]$
 $= \$(12\ 100 - 3.1w)$
- (b) (i) $(12\ 100 - 3.1w)\left(1 + \frac{10}{100}\right) - 3w = 0$
 $13\ 310 - 3.41w - 3w = 0$
 $6.41w = 13\ 310$
 $w = 2076.443\ 058$
 $= 2076.44$ (correct to 2 d.p.)
- (ii) Total amount repaid = $\$2076.443\ 058 \times 6$
 $= \$12\ 458.658$
Total interest paid = $\$12\ 458.658 - \$10\ 000$
 $= \$2458.658$
Required percentage = $\frac{2458.658}{10\ 000} \times 100\%$
 $= 24.6\%$ (correct to 3 sig. fig.)
- (c) He repaid his loan with progressively higher annual repayments so that the overall total interest he has to pay is less than that if his annual repayments are the same and fixed.
25. Mrs Lee wants to find a formula to calculate the exact monthly amount that was spent on water services (\$ W) in her home from October 2007 to April 2008. She figured from the bills in October to December 2007 that
- the water usage ($V\ \text{m}^3$) is first multiplied to the tariff rate (\$ T per m^3),
 - the product in (i) is then multiplied by 1.3 and 1.07,
 - 0.3 V is then added to (ii) and
 - finally 6 is added to (iii) to arrive at W .

- (e) Height of the solid = 15 cm
 $2(x + 3) = 15$
 $x + 3 = 7.5$
 $x = 4.5$
 When $x = 4.5$,
 $\text{gradient} = \frac{220 - 80}{4.5 - 3.5}$
 $= 140$
- (f) (i) Volume of second solid = $10 \times 7 \times (40 - x)$
 $= 70 \times (40 - x)$
 $= (280 - 70x) \text{ cm}^3$
- (iii) From the graph, $x = 3$.
 (iv) Total surface area
 $= 2[7(4 - 3) + 7 \times 10 + 10(4 - 3)]$
 $= 174 \text{ cm}^2$

Third Specimen Exam

Paper 1

1. (a) If an integer N is rounded off to the nearest 10, it becomes 50.
 List all the possible values of N that are
 (i) even numbers,
 (ii) prime numbers.
- (b) Insert a pair of brackets on the left hand side of each of the following equality to make the equality correct.
 (i) $3 + 11 \times 7 - 5 = 25$
 (ii) $28 - 4 + 28 \div 4 = 20$

Solution

- (a) (i) 46, 48, 50, 52, 54
 (ii) 47, 53
 (b) (i) $3 + 11 \times (7 - 5) = 25$
 (ii) $28 - (4 + 28) \div 4 = 20$
2. The speed of light is approximately 3×10^8 m/s.
 (a) Calculate the time taken, in seconds, for light to travel 2 km. Give your answer in standard form correct to 3 significant figures.
 (b) Express the answer in (a) correct to the nearest 100 nanosecond.

Solution

- (a) Time taken = $\frac{2000}{3 \times 10^8}$
 $= 6.67 \times 10^{-6} \text{ s}$ (correct to 3 sig. fig.)
 (b) $6.67 \times 10^{-6} \text{ s} = 6.67 \times 10^{-6} \times 10^9 \text{ nanoseconds}$
 $= 6.67 \times 10^3 \text{ nanoseconds}$
 $= 6670 \text{ nanoseconds}$
 $= 6700 \text{ nanoseconds}$ (correct to the nearest 100 nanoseconds)

3. Solve the following equations.

- (a) $\sqrt{2^x} = 64$
 (b) $5^{8+x} = 25^6$
 (c) $9^{2x+1} \times 27^{-x} = \left(\frac{1}{3}\right)^5$

Solution

- (a) $\sqrt{2^x} = 64$
 $2^{\frac{x}{2}} = 2^6$
 $\frac{x}{2} = 6$
 $x = 12$
- (b) $5^{8+x} = 25^6$
 $5^{8+x} = (5^2)^6$
 $8 + x = 12$
 $x = 4$
- (c) $9^{2x+1} \times 27^{-x} = \left(\frac{1}{3}\right)^5$
 $3^{2(2x+1)} \times 3^{3(-x)} = 3^{-5}$
 $4x + 2 - 3x = -5$
 $x = -7$

4. Khaleef and Jun Liang ran separately for the same distance to determine who is the faster sprinter. Suppose that Khaleef's time subtracted from Jun Liang's time is $(3xy - 18 - 6y + 9x)$ seconds.
 (a) Factorise completely $3xy - 6y - 18 + 9x$.
 (b) Who is the faster sprinter if y is positive and x is less than 2? Explain your answer.

Solution

- (a) $3xy - 6y - 18 + 9x = 3xy - 6y + 9x - 18$
 $= 3y(x - 2) + 9(x - 2)$
 $= (3y + 9)(x - 2)$
 $= 3(y + 3)(x - 2)$
- (b) If $y > 0$ and $x < 2$, then $3(y + 3)(x - 2) < 0$.
 \therefore time taken by Jun Liang is shorter.
 Jun Liang is the faster sprinter.

5. For each person, the proportion of a painting job that can be completed is directly proportional to the time spent. Working on their own, Hee Ming and Shahril can finish the painting job in 5 hours and 7 hours respectively.
 (a) How much time is needed to complete the painting job if Hee Ming and Shahril work together? Give your answer in hours and minutes.
 (b) Suppose that Hee Ming left after working with Shahril on the paint job for 1 hour. How much longer must Shahril spend on the paint job if he wants to complete it? Give your answer in hours and minutes.

Solution

- (a) In 1 h,
 Hee Ming can complete $\frac{1}{5}$ of the painting job,
 Shahril can complete $\frac{1}{7}$ of the painting job.
 If Hee Ming and Shahril work together, they can complete
 $\frac{1}{5} + \frac{1}{7} = \frac{12}{35}$ of the painting job in 1 h.
 Hence, Hee Ming and Shahril can complete the painting job in $\frac{35}{12}$ h i.e. 2 h 55 min if they work together.