

Lesson 8

Alien measure of angles or the mystery of agent 0.017

Radian measure of angles

- ✓ *Definition of a radian*
- ✓ *Expression of the length of a circle arc through the central angle and the radius*
- ✓ *Relation between degrees and radians*

Lesson 8

Nick: Dad, we always measured angles in degrees. I wonder, how do people on other planets measure angles?

Michelle: What a question, Nick! Think about it, who could know?

Dad: Actually, I really like this question. You know, guys, such questions may seem “dumb” at first, but sometimes they lead to very important discoveries. In fact, Nick’s question is reduced to this: “is there a somewhat natural measure of angles?” Certainly so far nobody knows how the aliens measure angles, but I’m 99.9% sure that it’s not in degrees.

Michelle: Why not?

Dad: Because a degree is an artificial measure that arose historically and is not based on the laws of nature. What is one degree?

Michelle: It’s $1/360$ part of a circle.

Dad: But where did the number 360 come from?

Michelle: I don’t know.

Dad: This number was introduced by astronomers in ancient Babylon (at least 3000 B.C.). No one knows for sure why they settled for this number. At those times, it was already known that the yearly cycle consists of 365 and $1/4$ days, even though astronomers didn’t know yet that the earth revolves around the sun.

Michelle: Maybe they just rounded 365 and $1/4$ to 360 and decided to use one “day” sizing $1/360$ part as a unit measure for angles?

Dad: Maybe.

Nick: But why didn’t they take 365? How is it worse?

Dad: Perhaps they chose 360 because it has more divisors. In other words, the number 360 splits into whole parts much better than 365. You can verify it yourselves. In any case, it’s clear that angle measure based on the number 360 is artificial. The same thing, by the way, applies to the decimal number system. It was formed only because we have 10 fingers on our hands. In math, and especially in computers, it is often more convenient to use other number systems such as binary or octal, in which the bases are powers of two.

N: So, is there a “natural” measure of angles?

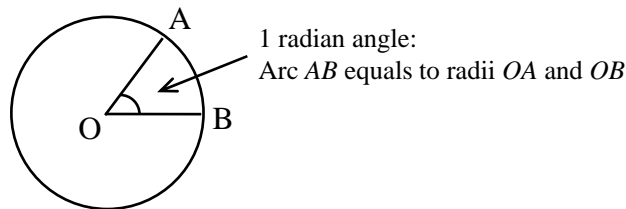
D: Yes, there is. And that measure is called a **radian**. I think aliens might use it since like us they study the same nature.

M: Is radian measure difficult to define?

D: No, pretty simple. Briefly, one radian measures an angle in an equilateral curvilinear triangle.

M: Wait a second, Dad. I understood nothing. What kind of a triangle is that? And why can't we use a regular equilateral triangle?

D: Well, take a look at this figure



Equilateral curvilinear triangle is just a sector AOB , in which the length of the arc AB is equal to the radius: $AB = OA = OB$. In such a sector, we accept by definition that the central angle AOB has a value of one radian.

M: Okay, but still it is not clear, why you claim this measure to be a natural one.

D: To understand the benefit of radian measure, let's consider the following problem...