Chapter 2 Multiplication and Division of Fractions

Students learned to multiply a fraction and a whole number in *Primary Mathematics (Standards Edition) Grade 4* and a fraction and another fraction in *Primary Mathematics (Standards Edition) Grade 5*. Division of a fraction by a whole number, a whole number and a fraction and a fraction by another fraction are taught in *Primary Mathematics (Standards Edition) Grade 5*. In this chapter, students complete their learning of division and multiplication of fractions by extending the procedure to mixed numbers.

To begin with, we review multiplication and division of fractions and deepen students' understanding of these operations. All number types are included – for multiplication, fraction multiplied by fraction, fraction multiplied by mixed number, and for division, fraction divided by whole number, whole number divided by fraction, fraction divided by fraction and mixed number divided by fraction. In each task, word problems are used to present concrete situations to give meaning to the multiplication or division procedures. Pictorial representations, including bar models are used to explain the operations.

For multiplication, the main idea is that one of the fractions is the 'multiplier' and the other represents a quantity. For example, in the first teaching activity (p. 58), $\frac{1}{3}$ is the 'multiplier' and $\frac{3}{4}$ represents the quantity of a pizza. In the second teaching activity (p. 59), the 'multiplier' is $\frac{1}{3}$ and the quantity is a mixed number representing distance. In the third teaching activity (p. 62), the 'multiplier' is a mixed number and the quantity is the volume of acid.

The first teaching activity (p. 58) helps students review the multiplication of a fraction and another fraction. The second teaching activity (p. 59) shows two ways to multiply a fraction and a mixed number. Method 1 uses the distributive property that $a(b + c) = ab + ac$. Thus, $3\frac{3}{4}$ is seen as $3 + \frac{3}{4}$. Method 2 converts the mixed number into improper fractions. Thus, $3\frac{3}{4}$ is seen as 15 fourths. The third teaching activity (p. 62) uses the distributive property that $(a + b)c = ac + bc$. Thus, $1\frac{1}{2}$ is seen as $1 + \frac{1}{2}$.

Consolidation tasks 2 and 3 (p. 60) help students move from multiplying a unit fraction to a non-unit fraction. Tasks 4 and 5 (p. 61) are further independent practice tasks based on the previous two tasks. Finally, in task 6 (p. 61), students are expected to perform the abstract procedure in the absence of any context. Note that the three questions in task 6 are of different levels of challenge.

In the rest of the section on multiplication, the consolidation tasks extend the idea of multiplication of fractions to algebraic expressions and provide opportunities for students to explain their thought processes.

In the subsequent lesson, two meanings of division are used to explain division of a fraction by a whole number (division as sharing or partitive division) and the division of a fraction/whole number by a fraction (division as grouping or quotitive division). Recall that these meanings have been introduced in *Primary Mathematics (Standards Edition) Grade* 1. Division as sharing is used to explain divisions such as $\frac{2}{3} \div 4$ while division as grouping is used to explain divisions such as

Assess	Have students solve **task 15, Textbook p. 67**. This is similar to task 14, except that the whole is 20 parts, since the denominator of the divisor is 20.	$\frac{9}{10} \div \frac{1}{20} = \frac{9}{10} \times \frac{20}{1} = 18$
Discuss division of a mixed number by a fraction	This section extends division to dividing mixed numbers by a fraction.	
	Discuss **task 16, Textbook p. 68**. Using the bar model in the textbook, 1 unit represents 1 m. Since we require $\frac{1}{2}$-m pieces, we divide each unit into halves. From the bar model, we can see that the string is made up of 7 units.	1 whole → 2 halves $3\frac{1}{2}$ wholes → $3\frac{1}{2} \times 2$ halves Therefore, $3\frac{1}{2} \div \frac{1}{2} = 3\frac{1}{2} \times 2$ $= \frac{7}{2} \times 2 = 7$.
Discuss division of a mixed number by a non-unit fraction	This section extends from the previous one to include division of a mixed number by a non-unit fraction.	
	Discuss **task 17, Textbook p. 68**. Use the first bar model in the textbook to explain to students that 1 unit represents 1 kg. Hence, $2\frac{1}{2}$ units represent $2\frac{1}{2}$ kg. Since we need to divide the flour into $\frac{5}{12}$ kg packets, we can divide each unit (1 kg) into 12 parts. This gives us the second model, with a total of 30 smaller units, with each small unit representing $\frac{1}{12}$ kg. Using ideas developed in the previous two tasks (tasks 15-16), it can be seen that $2\frac{1}{2}$ kg is made up of 30 twelfths. Students can then reason how many groups of 5 twelfths there are in $2\frac{1}{2}$ kg. Alternatively, challenge students to see that there are $2\frac{2}{5}$ of $\frac{5}{12}$-kg in 1 kg. Thus, there are $2\frac{1}{2} \times 2\frac{2}{5}$ of $\frac{5}{12}$-kg in $2\frac{1}{2}$ kg.	1 whole → 12 twelfths $2\frac{1}{2}$ → 30 twelfths 30 twelfths = 6 5-twelfths 1 kg → $2\frac{2}{5}$ of $\frac{5}{12}$ kg $2\frac{1}{2}$ kg → $2\frac{1}{2} \times 2\frac{2}{5}$ Therefore, $2\frac{1}{2} \div \frac{5}{12} = 2\frac{1}{2} \times \frac{12}{5}$ $= \frac{5}{2} \times \frac{12}{5}$ $= 6$.
Assess	Have students solve **tasks 18-20 on Textbook p. 69**.	Textbook p. 69 18. 7; $\frac{1}{3}$ 19. $2\frac{1}{2}$ 20. Answers may vary. Possible explanation: From diagram A, we can see that two-thirds of a whole is shaded. By

	Have students solve **tasks 1-3, Textbook p. 114-116**. Check their understanding of the concept learned. Task 2 Explain that we can also use the unitary method to solve the problems. Note that in this case, the proportion of the concentrate and water do not change. Task 3 (b) Point out that the units of measurement are different. Students should convert 12.5 m to 1250 cm before they perform the calculation.	Textbook p. 114 1. (a) 6, 9 (b) $\frac{1}{2}$, $\frac{1}{2}$, 1, $\frac{1}{2}$, $1\frac{1}{2}$ 2. (a) 80 ml (b) 0.2 l (c) $\frac{x}{9}$ l 3. (a) 250, 3750 cm, 37.5 (b) $\frac{1}{250}$, 5 cm, 5
Using different methods to solve proportion problems	Have students refer to **task 4, Textbook p. 117**. Explain to students that when a picture is *enlarged*, the size of the picture increases proportionately. Method 1: Interpret that since the building in the second picture is an enlarged version of the first, the corresponding sides of the buildings in the two pictures would be in the same ratio. From the model, we know that the height of the building in the original and enlarged picture can be represented by 4 and 5 units respectively. Combining the steps of dividing 12 cm by 4 to find the value of one unit and then multiplying it by 5 (to find the height of the building in the enlarged picture), we have $\frac{12}{4} \times 5$. Method 2: Interpret that since the building in the second picture is an enlarged version of the first, the corresponding sides of the buildings in the two pictures would be in the same ratio.	$12 \div 4 = 3$ $3 \times 5 = 15$ $\frac{12}{4} \times 5 = 15$